NORTH SYDNEY GIRLS HIGH SCHOOL

HSC Mathematics Extension 2

Assessment Task 2 Term 1 2015

Name: __________________ **Mathematics Class:** 12MZ__

Student Number: ________________

Time Allowed: **55 minutes + 2 minutes reading time**

Available Marks: **39**

Instructions:

- Questions are of equal value.
- Start each question in a new booklet.
- Show all necessary working.
- Do not work in columns.
- Marks may be deducted for incomplete or poorly arranged work.

Question	1-4	5	6a	6b	7a	7b	7c	Total
E4	/4			/7		/7		/18
E6		/1	/8		/4			/13
E9				/2			/6	/8
								/39

Section I

5 marks
Attempt Questions 1- 5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1- 5

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1 When $x^4 + x^3 - kx + 1$ is divided by $x^2 + 1$, the remainder is $3x + 2$. What is the value of k ?

(A) 4

(B) −2

(C) –4

(D) 2

2 A polynomial $P(x)$ is of degree 3 and it is known that $P'(a) = P'(b) = 0$ and $P(b) > P(a) > 0$. If $a < b$, then which of the following statements is true?

(A) The polynomial has 3 real zeros

(B) The polynomial has 1 real zero γ such that $\gamma < a$

(C) The polynomial has 1 real zero γ such that $a < \gamma < b$

(D) The polynomial has 1 real zero γ such that $\gamma > b$

3 Which of the following is the factorization of the polynomial $P(x) = x^3 + i$?

(A) $(x+i)(x^2 - ix + 1)$

(B) $(x-i)(x^2 + ix + 1)$

(C) $(x+i)(x^2 - ix - 1)$

(D) $(x-i)(x^2 + ix - 1)$

4 Given that $\dfrac{x-7}{x^2+x-6} \equiv \dfrac{A}{x-2} + \dfrac{B}{x+3}$, what are the values of A and B ?

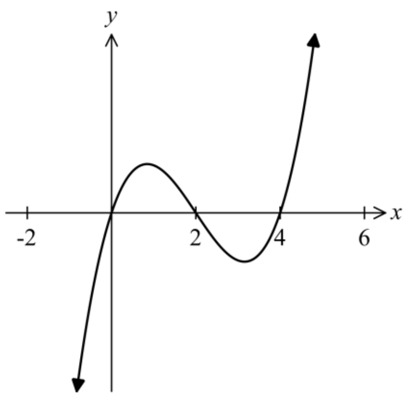
(A) $A = 2, \quad B = -1$

(B) $A = -1, \quad B = 2$

(C) $A = -2, \quad B = 1$

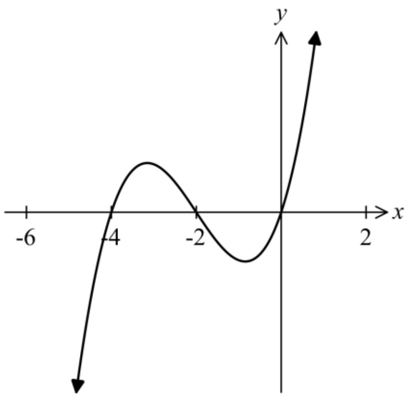
(D) $A = 1, \quad B = -2$

5 The graph of $y = f\left(x\right)$ is shown below.

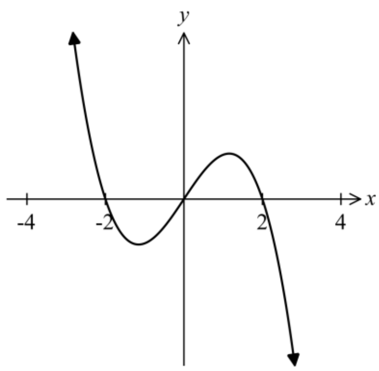


Which of the following could be the graph of $y = f\left(2-x\right)$?

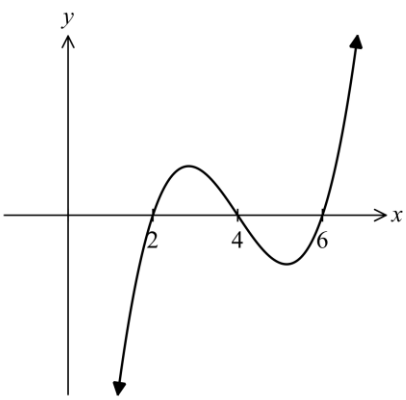
(A)



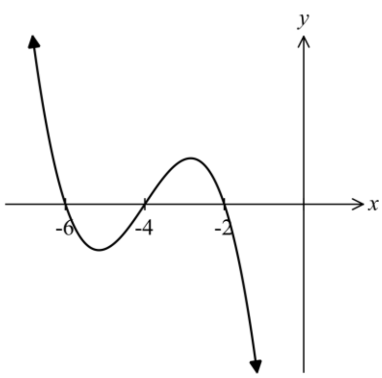
(B)



(C)



(D)



Section II

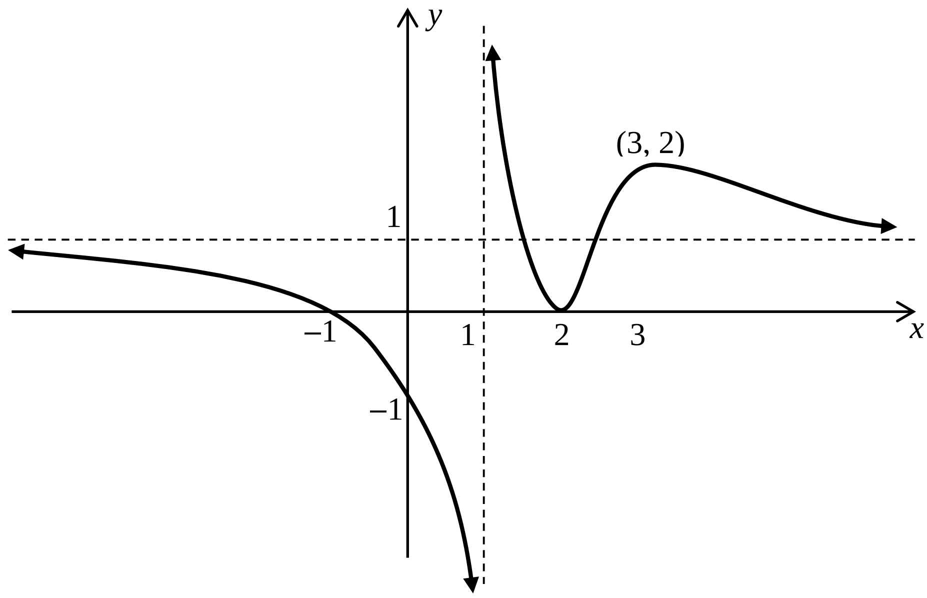
34 marks
Attempt Questions 6- 7
Allow about 47 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing paper is available.

In Questions 6 – 7, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (Use a SEPARATE writing booklet) **17 marks**

(a) Consider the graph of the function $y = f(x)$ shown below.



On the response sheets provided, sketch neat graphs of the following graphs, clearly showing the location of any turning points new asymptotes. If any portion of the original curve is part of your answer, you must clearly indicate this.

(i) $|y| = f(x)$ **2**

(ii) $y = \dfrac{1}{f(x)}$ **2**

(iii) $y = \sqrt{f(x)}$ **2**

(iv) $y = 2^{f(x)}$ **2**

- Question 6 continues on the next page -

Question 6 (continued)

(b) The equation $x^3 + ax^2 + bx + c = 0$, (a, b and c real), has three real roots α, β and γ.

(i) Find the following in terms of a, b and c.

1) $\alpha^2 + \beta^2 + \gamma^2$ **1**

2) $\alpha^3 + \beta^3 + \gamma^3$ **2**

(ii) Explain why $a^2 \geq 2b$. **1**

(iii) Find the cubic polynomial equation whose roots are α^2, β^2 and γ^2. **3**

(iv) Hence, show that $\dfrac{\alpha\beta}{\gamma} + \dfrac{\beta\gamma}{\alpha} + \dfrac{\gamma\alpha}{\beta} = 2a - \dfrac{b^2}{c}$. **2**

Question 7 (Use a SEPARATE writing booklet) **17 marks**

(a) Consider the curve $(x-y)^2 = x + y - 1$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{2x-2y-1}{2x-2y+1}$. **2**

(ii) Hence find the point(s) on the curve where the tangent is horizontal. **2**

(b) Consider the equation $z^4 - z^3 + z^2 - z + 1 = 0$.

(i) Show that the roots of this equation are $\text{cis}\left(\pm\dfrac{\pi}{5}\right)$ and $\text{cis}\left(\pm\dfrac{3\pi}{5}\right)$. **2**

(ii) Explain why the roots satisfy $z^2 - z + 1 - \dfrac{1}{z} + \dfrac{1}{z^2} = 0$. **1**

(iii) By letting $x = z + \dfrac{1}{z}$, obtain a quadratic equation in x. **2**

(iv) Hence, find the exact value of $\cos\left(\dfrac{3\pi}{5}\right)$. **2**

- Question 7 continues on the next page -

Question 7 (continued)

(c) The equation $px^3 + qx + r = 0$ has a double root.

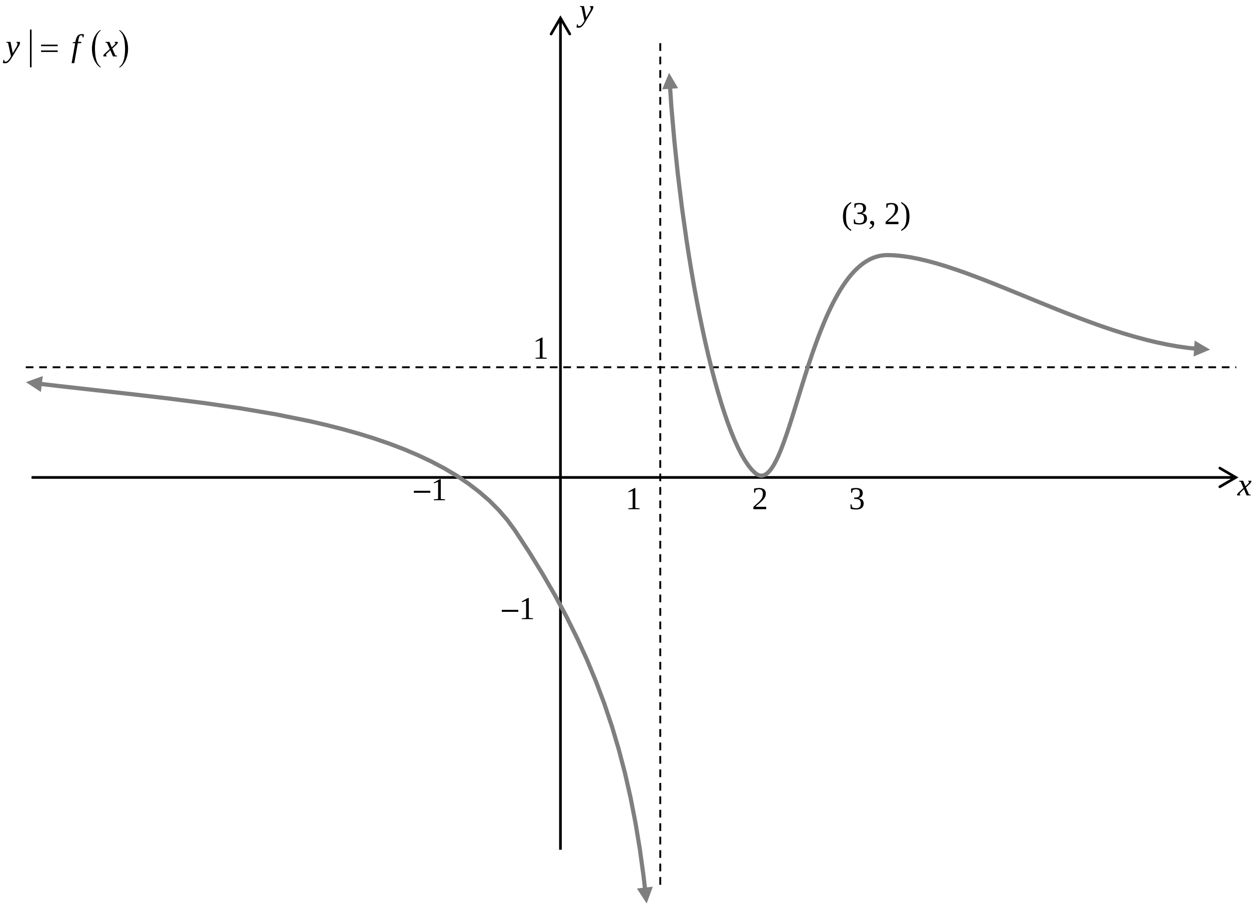
(i) Show that $27pr^2 + 4q^3 = 0$. **3**

(ii) Hence, find the exact value of the y-coordinates of the stationary points on the curve $y = 2x^3 - 3x + 1$ without the use of calculus. **3**

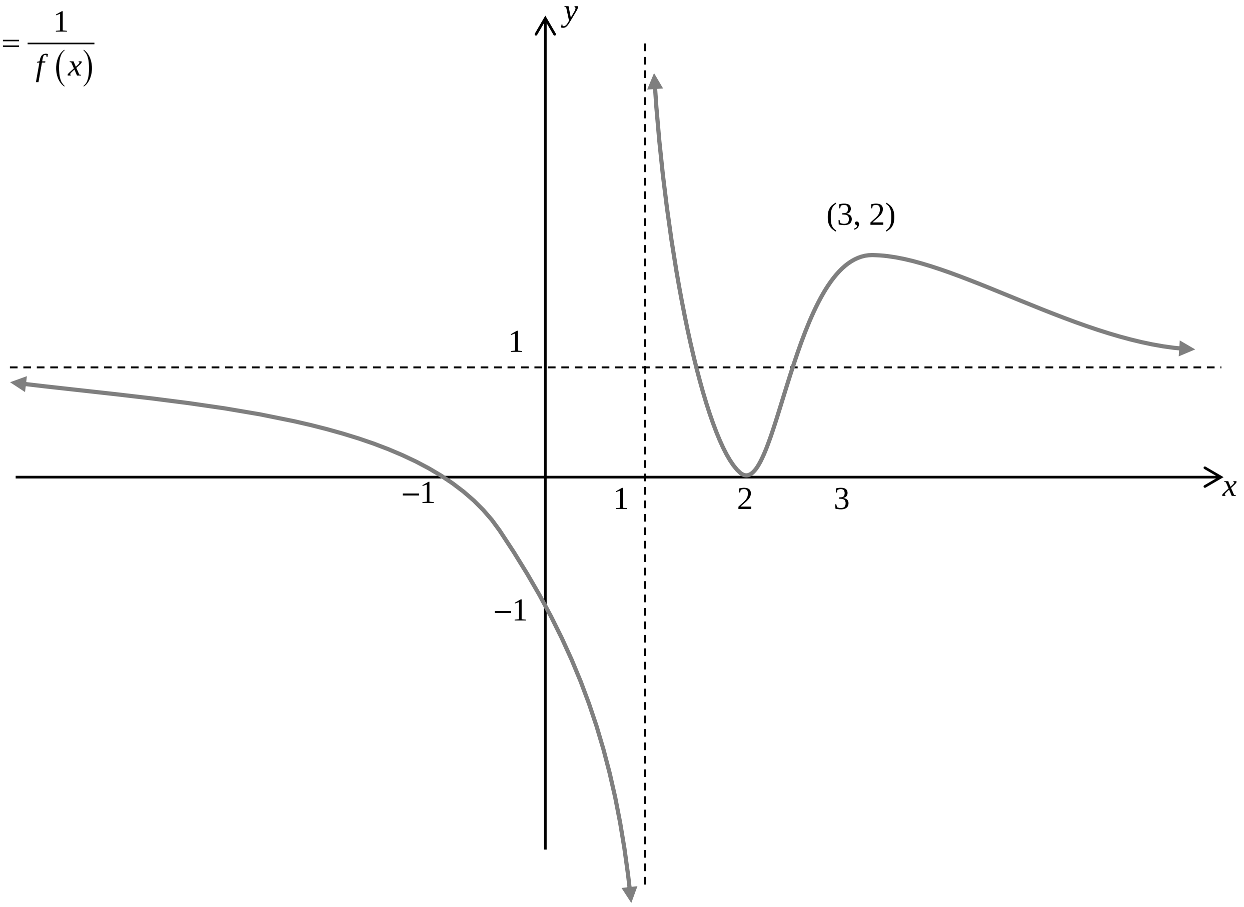
No credit will be awarded for finding the stationary points using calculus.

End of Test

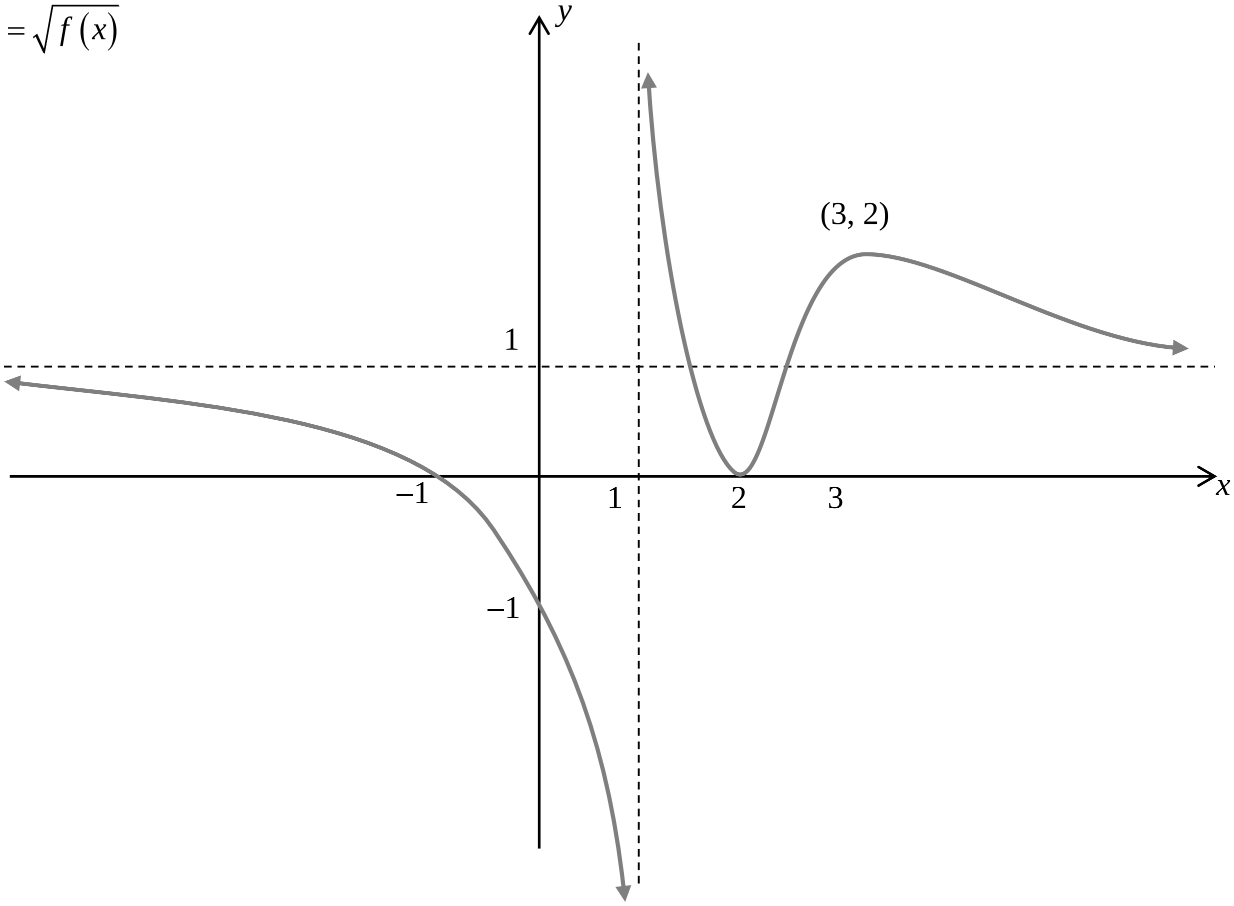
$|y| = f(x)$



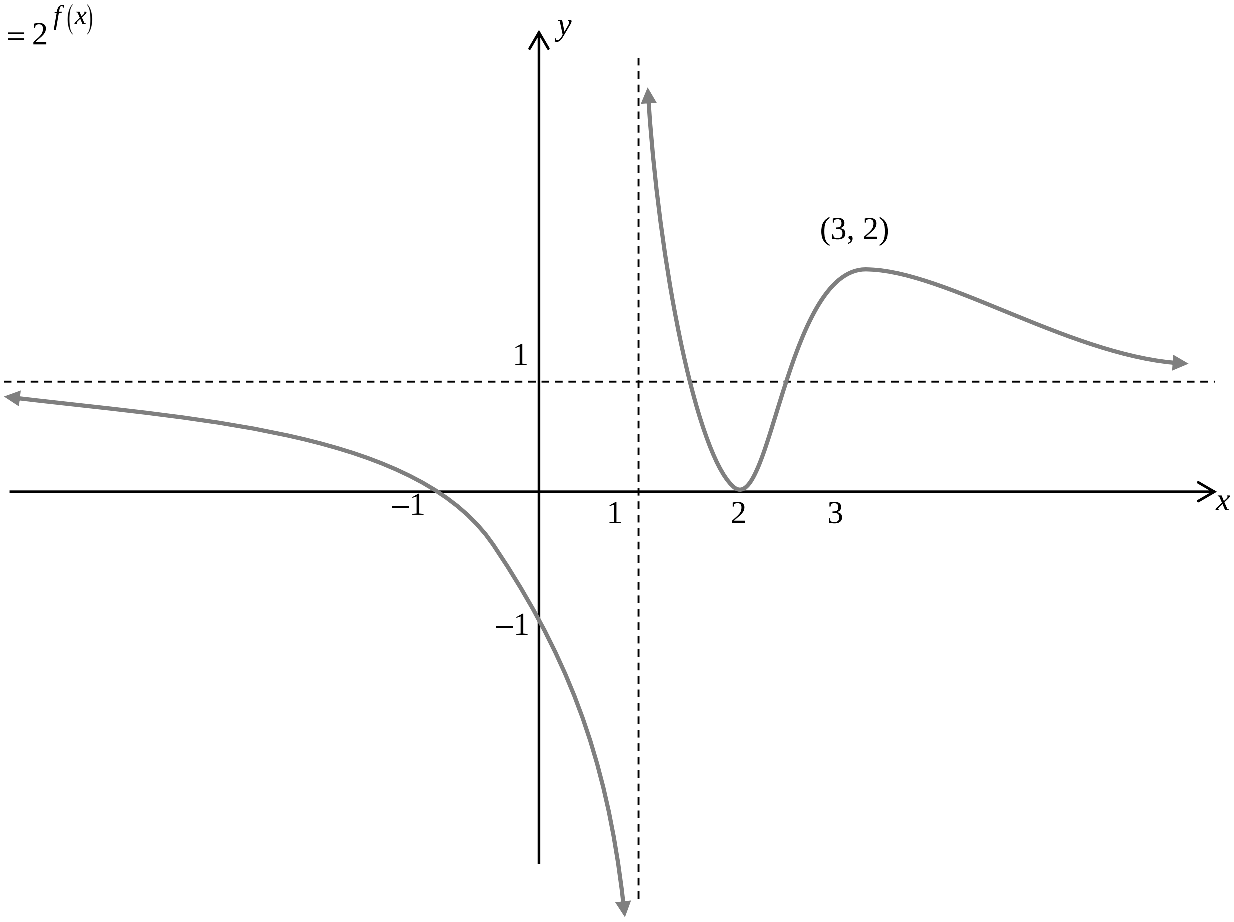
$y = \dfrac{1}{f(x)}$



$y = \sqrt{f(x)}$



$y = 2^{f(x)}$



NSGHS Extension 2 Task 2 2015 – Suggested Solutions and Markers Comments

Section I

1. C

Using the division transformation, $x^4+x^3-kx+1=\left(x^2+1\right)Q\left(x\right)+3x+2$

Sub $x=i$

$i^4-i^3-ki+1=3i+2$

$2-\left(k+1\right)i=2+3i$

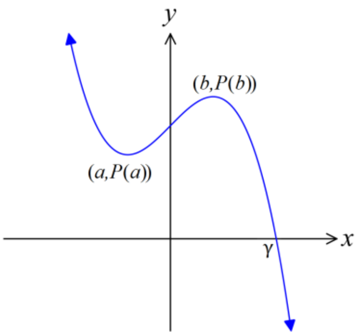
Equating imaginary parts:

$k+1=-3$

$k=-4$

2. D

Sketching the information provided, we need turning points at $x=a$ and $x=b$ with $P(b)>P(a)>0$.



3. D

$x^3+i=x^3-i^3$

$=\left(x-i\right)\left(x^2+ix+i^2\right)$

$=\left(x-i\right)\left(x^2+ix-1\right)$

4. B

$$\frac{x-7}{x^2+x-6}\equiv\frac{A\left(x+3\right)+B\left(x-2\right)}{\left(x-2\right)\left(x+3\right)}$$

$x-7=A\left(x+3\right)+B\left(x-2\right)$

Sub $x=-3\Rightarrow -5B=-10 \ \Rightarrow B=2$

Sub $x=2\Rightarrow 5A=-5 \ \Rightarrow A=-1$

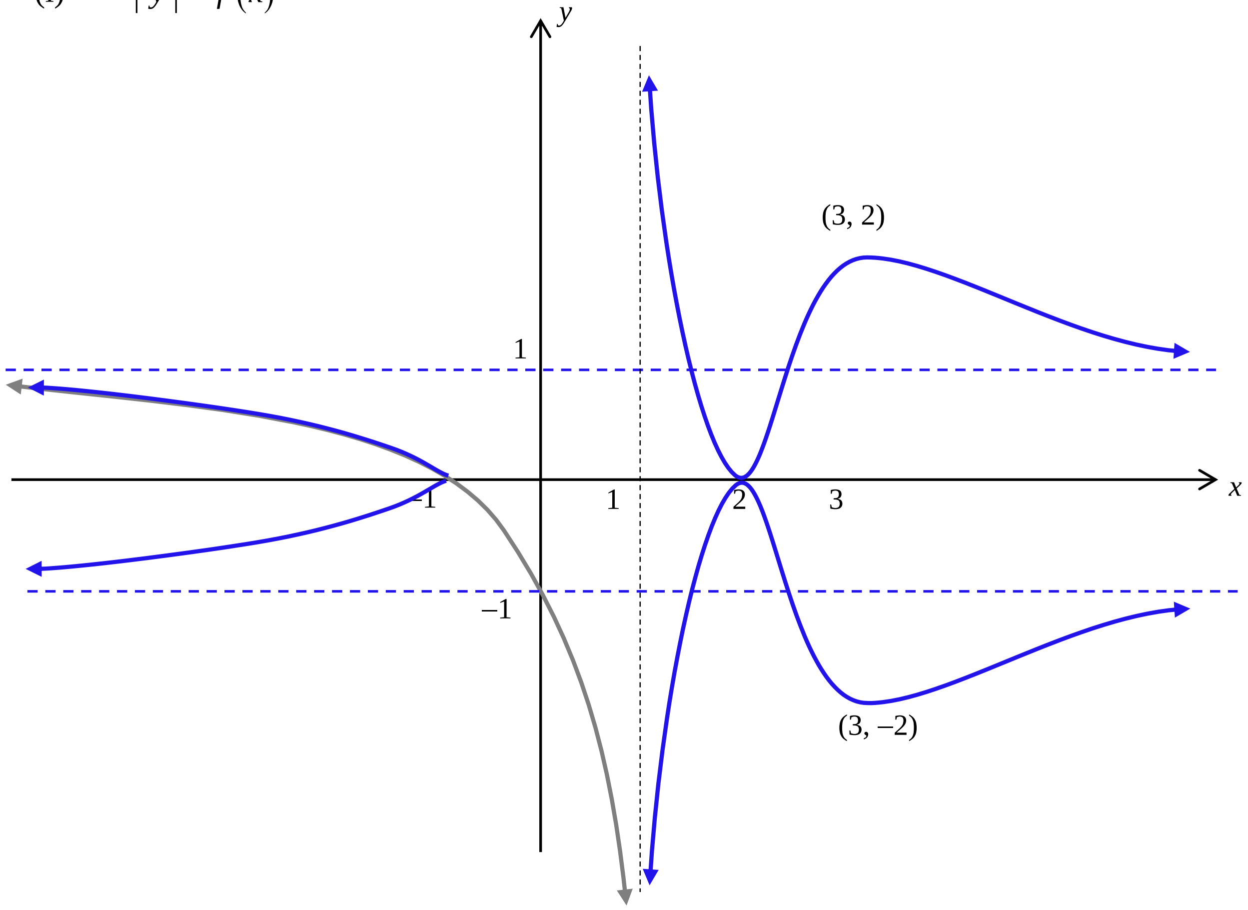
5. B

The graph of $y=f\left(2-x\right)$ is the reflection of the graph $y=f\left(x\right)$ in the line $x=1$.

Section II

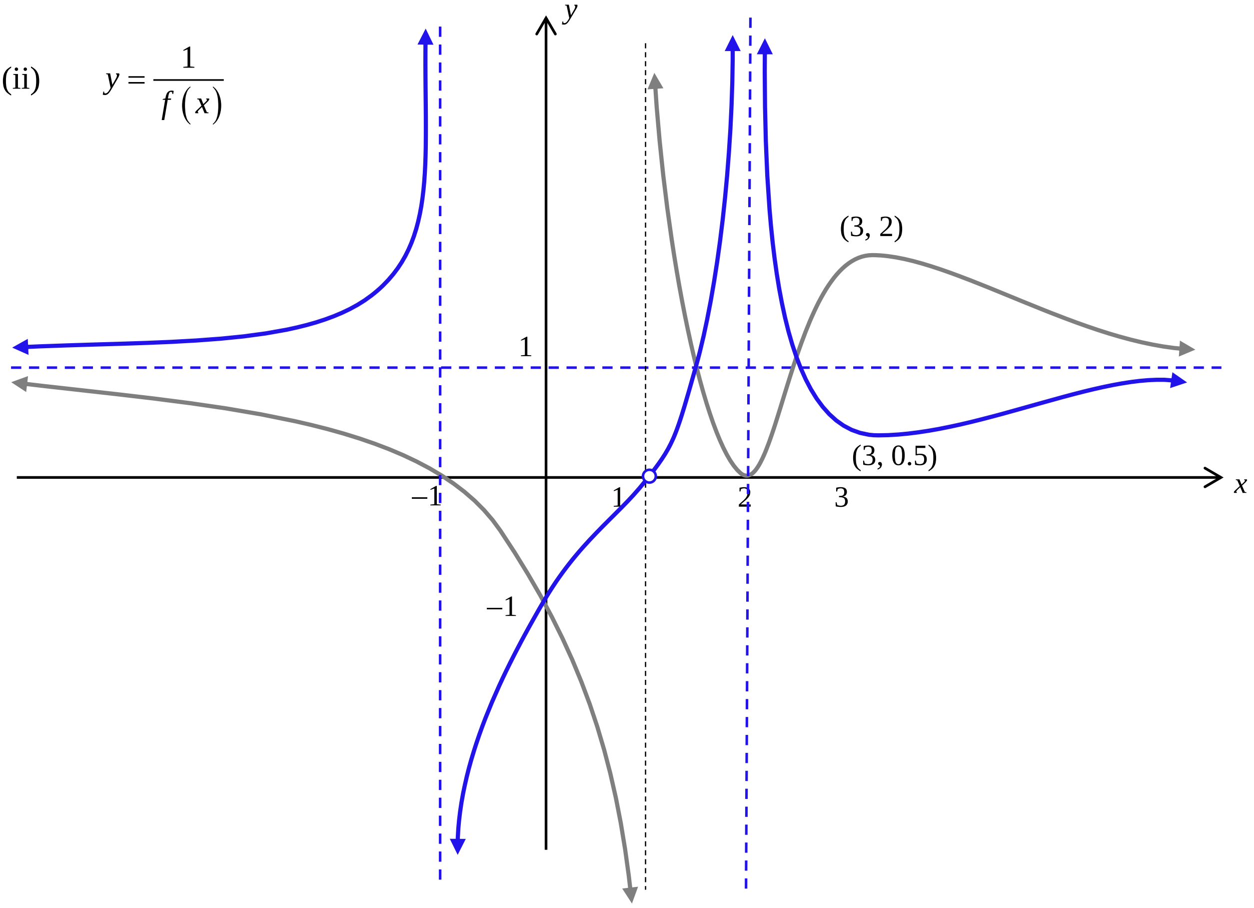
Question 6 (a)

(i) $|y| = f(x)$



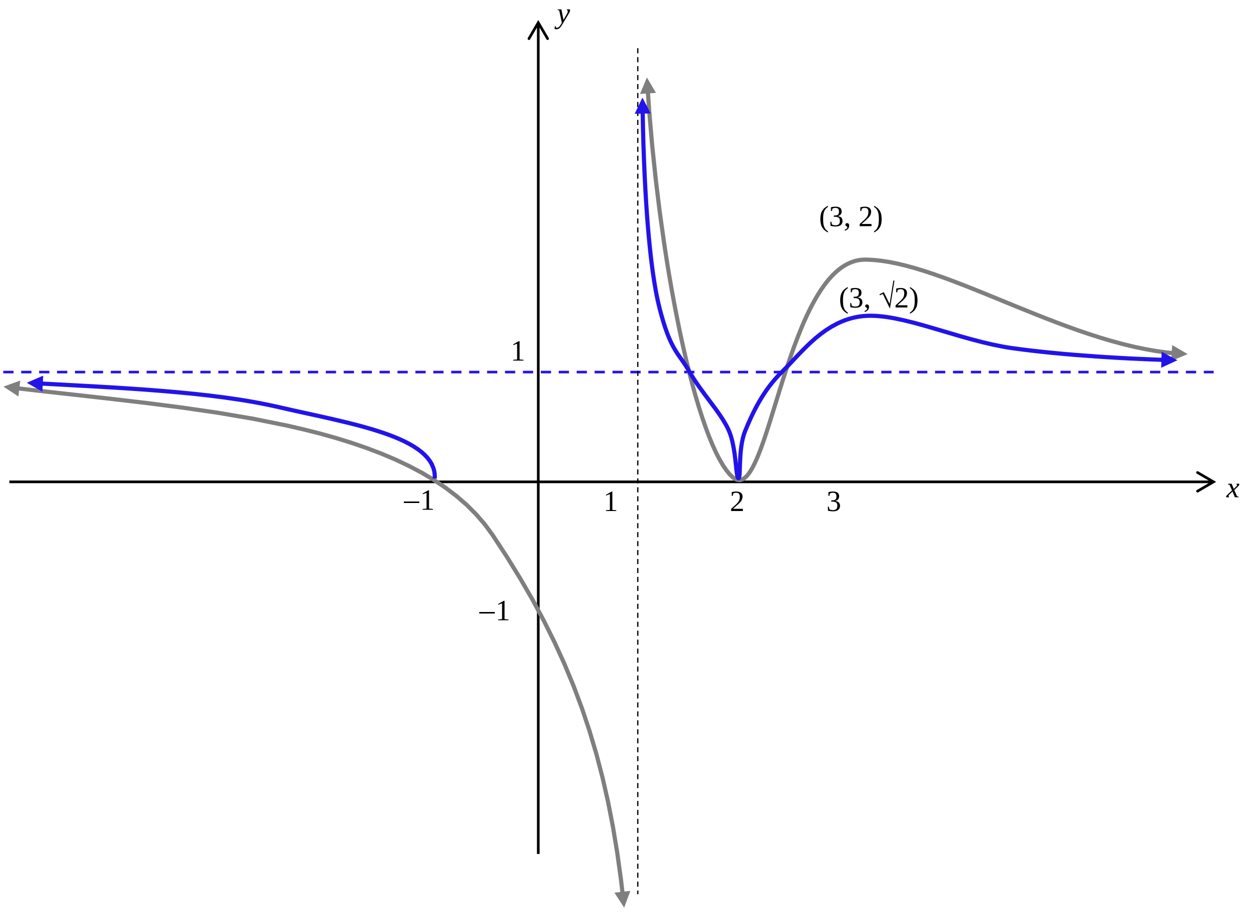
(a)	Many students did not follow instructions. You were asked to use the supplied response sheet.
(i)	A large number of students drew $y = \left| f(x) \right|$. This earned zero marks.
	Other students did not indicate that they were including parts of the original curve, as instructed. I had to assume that you did not know these sections were meant to be included in the answer.
	Many students did not exclude the part of the graph where $f(x)$ is negative. Others did not indicate the coordinates of the new turning point.

(ii) $y = \dfrac{1}{f(x)}$



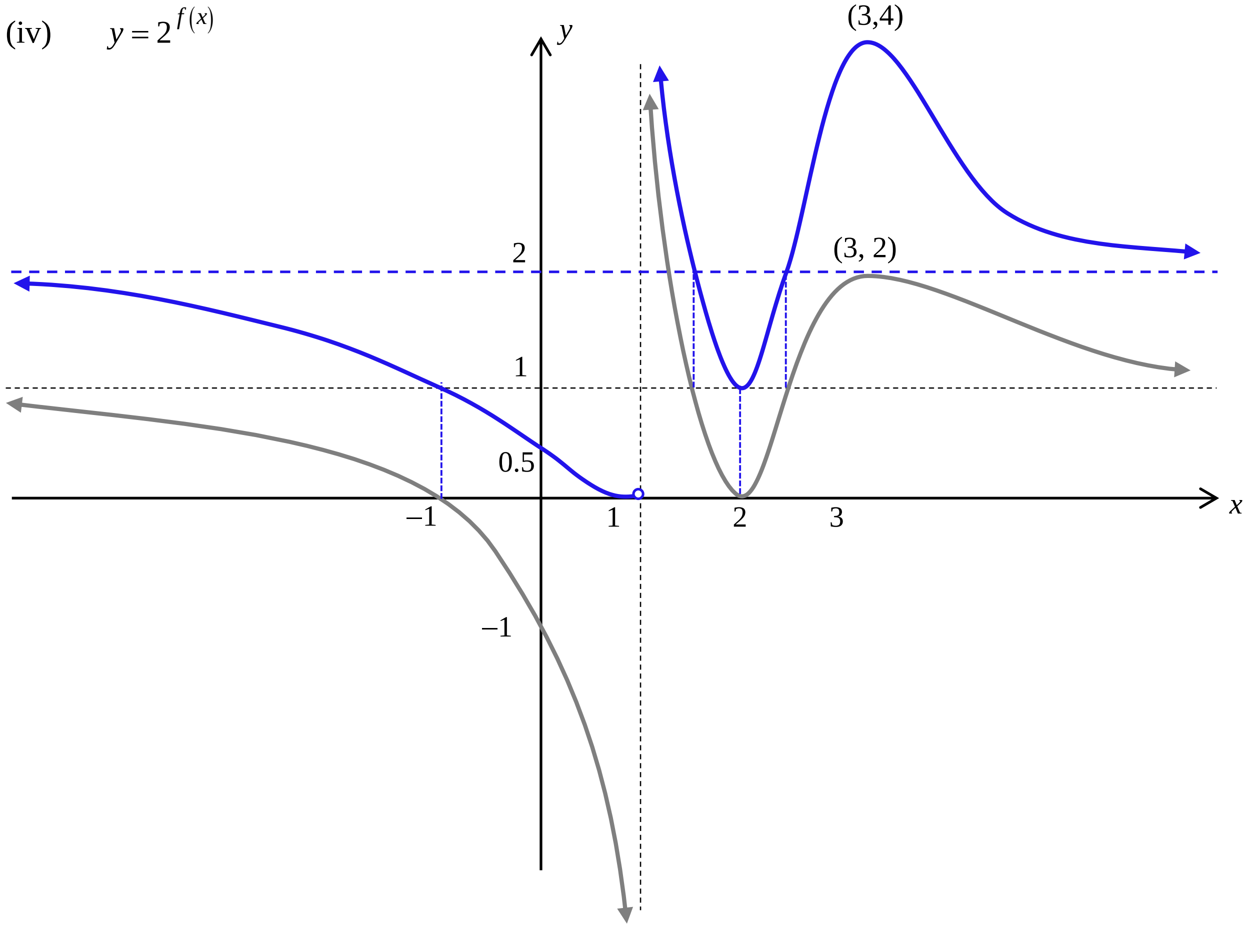
(ii)	A number of students didn't grasp the basic concepts here, and some responses looked quite randomly drawn around the middle section of the graph. Again, the y-value of the new turning point has to be indicated. A number of students wrote $\frac{1}{3}$ instead of $\frac{1}{2}$ - we are reciprocating y-values, not x-values. Also, the new stationary point and the original stationary point must line up vertically. The reciprocal of 1 is 1; the reciprocal of -1 is -1. So the new and old graphs should cross on the horizontal asymptote and at the y-intercept.

(iii) $y = \sqrt{f(x)}$



(iii)	Your graph should have a vertical tangent at $x = -1$. Yet again, many students left off the coordinates of the new turning point, or didn't align it vertically with the original turning point. The graphs should cross on the line $y = 1$, so students who answered in the writing booklet without drawing the original graph were automatically penalised.

(iv) $y = 2^{f(x)}$



(iv) The biggest issue was for $-1 < x < 1$. $\lim_{x \to -\infty} 2^x = 0$, so the curve should approach an open circle at $(1, 0)$. An exponential function of the form $y = a^{f(x)}$ can never produce negative y-values. Again, many students used the x-value instead of the y-value to calculate the new y-value of the stationary point.

$2^{f(x)}$ is always greater than $f(x)$, so the graphs should never cross.

Question 6 (b)

(i) Using the relationship between roots and coefficients:

$\alpha+\beta+\gamma=-a$

$\alpha\beta+\beta\gamma+\alpha\gamma=b$

$\alpha\beta\gamma=-c$

1) $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)=(-a)^2-2(b)=a^2-2b$

(1) Well done, though a number of students spent time developing the equation with roots α^2, β^2 and γ^2 in order to read off the sum of the roots. Had this not been required in a later part, this would have been a time waster ... it is much quicker to use $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$.

2) $\alpha^3+a\alpha^2+b\alpha+c=0$ ($\because \alpha$ is a root)

Similarly, $\beta^3+a\beta^2+b\beta+c=0$ and $\gamma^3+a\gamma^2+b\gamma+c=0$

Adding the three equations,

$(\alpha^3+\beta^3+\gamma^3)+a(\alpha^2+\beta^2+\gamma^2)+b(\alpha+\beta+\gamma)+3c=0$

$(\alpha^3+\beta^3+\gamma^3)+a(a^2-2b)+b(-a)+3c=0$

$\alpha^3+\beta^3+\gamma^3=-a^3+3ab-3c$

(2) You needed to **develop** the result by substitution followed by addition.

Students who wrote $\sum\alpha^3 = -a\sum\alpha^2-b\sum\alpha-3c$ without development were penalised.

A few students thought that the final term should follow the same pattern as the first two: $c+c+c=3c$!

(ii) $\alpha^2+\beta^2+\gamma^2\ge 0$ as α,β,γ are real

$\therefore a^2-2b\ge 0$ (from (i) part 1)

i.e. $a^2\ge 2b$

(ii) Saying $\alpha^2+\beta^2+\gamma^2\ge 0$ was not enough. It is true only because the roots are real ... this had to be stated.

(iii) The equation whose roots are $\alpha^2, \beta^2, \gamma^2$ is given by:

$$\left(\sqrt{x}\right)^3 + a\left(\sqrt{x}\right)^2 + b\left(\sqrt{x}\right) + c = 0$$

$$x\sqrt{x} + ax + b\sqrt{x} + c = 0$$

$$\sqrt{x}\left(x+b\right) = -\left(ax+c\right) \qquad \text{square both sides}$$

$$x\left(x+b\right)^2 = \left(ax+c\right)^2$$

$$x\left(x^2+2bx+b^2\right) = a^2x^2+2acx+c^2$$

$$x^3+\left(2b-a^2\right)x^2+\left(b^2-2ac\right)x-c^2=0$$

(iii) Those students who had already done this to answer (i) (1) were fine, provided they referred back to that question. This question was well done.

(iv) $$\frac{\alpha\beta}{\gamma}+\frac{\beta\gamma}{\alpha}+\frac{\gamma\alpha}{\beta}=\frac{\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2}{\alpha\beta\gamma}$$

$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2 = b^2-2ac$ using sum of pairs of roots of equation in (iii)

$$\frac{\alpha\beta}{\gamma}+\frac{\beta\gamma}{\alpha}+\frac{\gamma\alpha}{\beta}=\frac{b^2-2ac}{-c}=\frac{2ac-b^2}{c}=2a-\frac{b^2}{c}.$$

(iv) The question said **HENCE**. It did not give you an ‘otherwise’ option. Students who did not use the result of part (iii) received at most 1/2. In any case, many students who tried to come up with a formula for $\alpha^2\beta^2+\alpha^2\gamma^2+\beta^2\gamma^2$ by expanding $\left(\alpha+\beta+\gamma\right)^3$ were not successful, or did not show enough development of the result. This is a **SHOW** question, so you must clearly develop the given result, explaining all steps where necessary. Saying ‘from part (iii)’ was not sufficient’. You had to state ‘taking the sum of roots in pairs from part (iii)’. Further, the sum of roots in the denominator comes from the **first** equation ... you had to indicate which equation you were drawing each result from. Many students stated $\alpha^2\beta^2\gamma^2 = c^2$ from the second equation, then concluded incorrectly that $\alpha\beta\gamma = c$. You needed the first equation to get the correct result for the product of roots.

Question 7 (a)

(i) Differentiating implicitly with respect to x

$$2(x-y)(1-y')=1+y'$$

$$2x-2xy'-2y+2yy'=1+y'$$

$$y'(2y-2x-1)=2y-2x+1$$

$$y'=\frac{2y-2x+1}{2y-2x-1}$$

$$\frac{dy}{dx}=\frac{2x-2y-1}{2x-2y+1}$$

7 (a) was generally very well done.

(i) Practise using the chain rule. To differentiate $(x-y)^2$, you <u>do not</u> need to expand. In this instance, it is not very tedious but would you really consider expanding $(x-y)^{10}$ in order to differentiate it? A small number of students have not learnt to differentiate implicitly.

(ii) For a horizontal tangent $\frac{dy}{dx}=0$

$$2x-2y-1=0$$

$$y=x-\tfrac{1}{2}$$

Sub into the equation of the curve.

$$\left(x-\left(x-\tfrac{1}{2}\right)\right)^2=x+x-\tfrac{1}{2}-1$$

$$\tfrac{1}{4}=2x-\tfrac{3}{2}$$

$$2x=\tfrac{7}{4}$$

$$x=\tfrac{7}{8}$$

When $x=\tfrac{7}{8}$, $y=\tfrac{7}{8}-\tfrac{1}{2}=\tfrac{3}{8}$

Therefore, the curve has a horizontal tangent at $\left(\tfrac{7}{8},\tfrac{3}{8}\right)$.

(ii) This part was very well done. Students generally knew what they needed to do.

A small number of students found the x- value and then subbed back into the original curve to find the y-value. You really should be using the relationship $y=x-\tfrac{1}{2}$ which is what you got as the condition for a horizontal tangent. In this question, it didn't matter much, as there is a unique y-value, although the algebra is more involved. However, in other cases there may be more than 1 point with the same x-value and only one of them may correspond to the horizontal tangent. Look at the last question in the 2014 Term 1 paper.

Question 7 (b)

(i) Consider $z^5+1=(z+1)(z^4-z^3+z^2-z+1)$

The roots of $z^5+1=0$ are the fifth roots of -1

i.e. $z=\left(\text{cis}(\pi+2k\pi)\right)^{\frac{1}{5}}$, $k=0,\pm1,\pm2$ by De Moivre's Theorem

$z=\text{cis}\left(\frac{\pi}{5}\right),\text{cis}\left(\frac{3\pi}{5}\right),\text{cis}\left(\frac{5\pi}{5}\right),\text{cis}\left(-\frac{\pi}{5}\right),\text{cis}\left(-\frac{3\pi}{5}\right)$ are the roots of $z^5+1=0$

$z^5+1=0$ implies $(z+1)=0$ or $(z^4-z^3+z^2-z+1)=0$

$z=\text{cis}\left(\frac{5\pi}{5}\right)=-1$ corresponds to $z+1=0$.

Therefore, the roots of $z^4-z^3+z^2-z+1=0$ are $z=\text{cis}\left(\pm\frac{\pi}{5}\right),\text{cis}\left(\pm\frac{3\pi}{5}\right)$

(i) Most students knew that they needed the fifth roots of -1 (A small number of students held fast to the notion that it was fifth roots of 1 they needed and somehow transformed these roots to the ones required in the question). However, it is one thing to know and quite another to communicate what it has to do with the roots of the given polynomial. Show the factorisation $z^5+1=(z+1)(z^4-z^3+z^2-z+1)$ – if you didn't or offered incorrect factorisation, there was a penalty. The roots of $z^5+1=0$ includes –1, which has to be discounted as it corresponds to $z+1=0$ and therefore the remaining four roots must correspond to $z^4-z^3+z^2-z+1=0$. If this was not communicated clearly, there was a penalty. When writing $\text{cis}\pm\frac{3\pi}{5}$, brackets are recommended ie write $\text{cis}\left(\pm\frac{3\pi}{5}\right)$.

(ii) Consider $z^4-z^3+z^2-z+1=z^2\left(z^2-z+1-\frac{1}{z}+\frac{1}{z^2}\right)$

$z^4-z^3+z^2-z+1=0$ implies $z=0$ or $z^2-z+1-\frac{1}{z}+\frac{1}{z^2}=0$

$z=0$ is not a root of $z^4-z^3+z^2-z+1=0$ (see part (i))

Thus, the roots of $z^4-z^3+z^2-z+1=0$ are also the roots of $z^2-z+1-\frac{1}{z}+\frac{1}{z^2}=0$

(ii) You are asked to "explain". Do not shy away from using words.

Many students were not able to distinguish between what they are allowed to assume and what they need to show. Some students stated that the roots of $z^2-z+1-\frac{1}{z}+\frac{1}{z^2}=0$ satisfy $z^4-z^3+z^2-z+1=0$ instead of the opposite. This was penalised.

Also, if $A(x)=B(x).C(x)$, then it is not evident or necessary that all the roots of $A(x)$ are also roots of $C(x)$, unless you know that $B(x)=0$ does not correspond to any of the roots of $A(x)$. So students who stated that $z^4-z^3+z^2-z+1=z^2\left(z^2-z+1-\frac{1}{z}+\frac{1}{z^2}\right)$ needed to also state that $z=0$ is not a root of $z^4-z^3+z^2-z+1=0$. Otherwise, there was a penalty.

(iii) $x = z + \frac{1}{z}$

$$x^2 = \left(z + \frac{1}{z}\right)^2 = z^2 + \frac{1}{z^2} + 2$$

$$z^2 + \frac{1}{z^2} = \left(z + \frac{1}{z}\right)^2 - 2 = x^2 - 2$$

Substituting in the equation $z^2 - z + 1 - \frac{1}{z} + \frac{1}{z^2} = 0$

$$\left(z^2 + \frac{1}{z^2}\right) - \left(z + \frac{1}{z}\right) + 1 = 0$$

$$x^2 - 2 - x + 1 = 0$$

$$x^2 - x - 1 = 0$$

(iii) This part was well done and most students were successful in obtaining the correct quadratic. A small number of students offered a quadratic expression instead of an equation and incurred a penalty.

(iv) The roots of $x^2 - x - 1 = 0$ correspond to the roots of $z^2 - z + 1 - \frac{1}{z} + \frac{1}{z^2} = 0$ where $x = z + \frac{1}{z}$

or $x = \text{cis}\,\theta + \frac{1}{\text{cis}\,\theta} = 2\cos\theta$.

Therefore, $x = 2\cos\left(\frac{\pi}{5}\right), 2\cos\left(\frac{3\pi}{5}\right)$. Note that $x = 2\cos\left(-\frac{\pi}{5}\right), 2\cos\left(-\frac{3\pi}{5}\right)$ do not yield new roots.

Using the quadratic formula, the roots of $x^2 - x - 1 = 0$ are $x = \frac{1 \pm \sqrt{5}}{2}$.

Therefore, $2\cos\left(\frac{3\pi}{5}\right) = \frac{1 - \sqrt{5}}{2}$ as $\cos\left(\frac{3\pi}{5}\right) < 0$ being a second quadrant angle.

And $\cos\left(\frac{3\pi}{5}\right) = \frac{1 - \sqrt{5}}{4}$.

(iv) Most students knew they had to solve the quadratic equation and obtain the roots $\frac{1 \pm \sqrt{5}}{2}$.

This was worth half a mark. You then needed to connect the roots of this equation to the roots of the original equation from part (i). Some students incorrectly assumed that the roots of the quadratic were the values of $\cos\frac{\pi}{5}$ and $\cos\frac{3\pi}{5}$ and could not earn the rest of the marks.

Those who successfully connected the roots to $z + \frac{1}{z}$ further needed to identify which of the two roots pertained to $\cos\frac{3\pi}{5}$ to earn the full credit.

This was a "Hence" question and not a "Hence, or otherwise". Students who used alternate methods to arrive at the correct value of $\cos\frac{3\pi}{5}$, sadly could not be awarded marks as they ignored the instructions in the question.

Question 7 (c)

(i) Consider $P(x) = px^3 + qx + r$

$P(x)$ has a double zero at $x = \alpha$. Then, $P(\alpha) = P'(\alpha) = 0$.

$P'(\alpha) = 0 \Rightarrow 3p\alpha^2 + q = 0$ or $\alpha^2 = -\dfrac{q}{3p}$ (1)

Also, $P(\alpha) = 0$

$px^3 + qx + r = 0$

$x\left(px^2 + q\right) = -r$ square both sides

$x^2\left(px^2 + q\right)^2 = r^2$

sub $\alpha^2 = -\dfrac{q}{3p}$ from (1)

$$\left(-\frac{q}{3p}\right)\left(p\left(-\frac{q}{3p}\right) + q\right)^2 = r^2$$

$$\left(-\frac{q}{3p}\right)\left(-\frac{2q}{3}\right)^2 = r^2$$

$$\left(-\frac{q}{3p}\right)\left(\frac{4q}{9}\right) = r^2$$

$$-\frac{4q^3}{27p} = r^2$$

$$-4q^3 = 27pr^2$$

$$27pr^2 + 4q^3 = 0$$

ALTERNATE APPROACH

Let the roots be α, α and β.

Sum of roots: $2\alpha + \beta = 0$ (1)

Sum of pairs: $\alpha^2 + 2\alpha\beta = \dfrac{q}{p}$ (2)

Product of roots: $\alpha^2\beta = -\dfrac{r}{p}$ (3)

From (1) $\beta = -2\alpha$ sub into (2) $\alpha^2 + 2\alpha(-2\alpha) = \dfrac{q}{p}$

$-3\alpha^2 = \dfrac{q}{p} \Rightarrow \alpha^2 = -\dfrac{q}{3p}$ (4)

Sub $\beta = -2\alpha$ into (3) $\alpha^2(-2\alpha) = -\dfrac{r}{p}$

$-2\alpha^3 = -\frac{r}{p} \Rightarrow \alpha^3 = \frac{r}{2p}$ (5)

From (4) $\alpha^6 = -\frac{q^3}{27p^3}$ and from (5) $\alpha^6 = \frac{r^2}{4p^2}$

Equating, $-\frac{q^3}{27p^3} = \frac{r^2}{4p^2}$ and $-4p^2q^3 = 27p^3r^2$

Dividing both sides by p^2 as $p \neq 0$ and rearranging $27pr^2 + 4q^3 = 0$

(i)	Many students state blindly that $P(x)=0$ and $P'(x)=0$ without any reference to a root or a double root. This incurred a small penalty. Better responses assumed the double root was α and then stated $P(\alpha)=P'(\alpha)=0$. There were several successful approaches. The easiest was to use the multiple root theorem but 3 unit methods of assuming the roots to be α , α and β and using relationships between roots and coefficients were also successful. The solutions detail a couple of the common approaches used. Please take care when writing q's that resemble 9's. Try to use distinctive ways of writing letters that can be mistaken for other things.

(ii) Let the *y*-coordinate of the stationary point of the curve $y = 2x^3 - 3x + 1$be *k*.

Then, the curve bounces off the line $y = k$ or the equation $2x^3 - 3x + 1 = k$ has a double root.

i.e. $2x^3 - 3x + 1 - k = 0$ has a double root.

$$27(2)(1-k)^2 + 4(-3)^3 = 0$$ From (i)

$$54(1-k)^2 = 108$$

$$(1-k)^2 = 2$$

$$1-k = \pm\sqrt{2}$$

$$k = 1 \pm \sqrt{2}$$

Therefore, the *y*-coordinates of the stationary points are $1+\sqrt{2}, 1-\sqrt{2}$

(ii)	Very few students made the connection with the previous part. Most students incorrectly assumed that the polynomial $2x^3 - 3x + 1 = 0$ had a double root and assigned values of $p = 2; q = -3$ and $r = 1$ and subbed this into the relationship shown in part (i). This results in $27(2)(1)^3 + 4(-3)^3 = 0$ which is clearly not true. Yet most students were unbothered by this and interpreted the resulting value of the expression –54 variously as the *x*-value of the double root or the *y*-value of the stationary point. Many students insisted that the stationary point corresponds to the double root. It does if the stationary point resides on the *x*-axis, but most stationary points on curves do not lie on the *x*-axis.

End of solutions